Since the velocity along the plane is not affected by the impacts the velocity v' at the end of time, t, will be upward along the plane end of magnitude

$$\begin{aligned} v' &= v \cos \varphi - g \sin \psi t \\ &= v \cos \varphi - \frac{2v \sin \varphi \tan \psi}{(1-e)}, \qquad \text{since} \qquad t = \frac{2v \sin \varphi}{(1-e) g \cos \psi}. \end{aligned}$$

Also solved by the PROPOSER.

335. Proposed by HAROLD T. DAVIS, Colorado Springs, Colorado.

A heavy particle is projected upwards with a velocity V in a medium resisting as the nth power of the velocity. Prove that the elevation of the particle when the velocity downwards is V is equal to LT, where L is the limiting velocity and T is the time in which the particle falling from rest in the medium will acquire a velocity V^2/L.

SOLUTION BY JOS. B. REYNOLDS, Lehigh University.

As the solution will show, this problem should read as follows: A heavy particle is projected upwards with a velocity V in a medium resisting as the nth power of the velocity. Prove that the whole space (up and down) described when the downward velocity is V is equal to LT, where L is the limiting velocity and T is the time in which the particle falling from rest in the medium will acquire a velocity V^2/L.

The equation of motion for the particle upward is

(1) $$-v\frac{dv}{ds} = g + \mu v^n$$

μ being the proportionality factor, and the equation of motion for the particle downward is

(2) $$v\frac{dv}{ds} = g - \mu v^n = \frac{dv}{dt}.$$

When the particle has acquired the limiting velocity L its acceleration is zero. So by equation (2) we have $g/\mu = L^n$. Also by (2), when falling in the medium,

$$dt = \frac{dv}{g - \mu v^n}, \qquad \text{and} \qquad \therefore \quad T = \int_0^{V^2/L} \frac{dv}{g - \mu v^n} = \frac{1}{\mu}\int_0^{V^2/L} \frac{dv}{L^n - v^n};$$

whence,

(3) $$LT = \frac{L^{n+1}}{g}\int_0^{V^2/L} \frac{dv}{L^n - v^n}.$$

From (1), in the upward motion,

$$ds = -\frac{vdv}{g + \mu v^n};$$

whence, if the particle rises to a height h before coming to rest,

$$h = -\int_V^0 \frac{vdv}{g + \mu v^n} = \int_0^V \frac{vdv}{g + \mu v^n}.$$

When falling, by (2),

$$ds = \int \frac{vdv}{g - \mu v^n},$$

so that if it has fallen a distance h_1 when the velocity is V, we have

$$h_1 = \int_0^V \frac{vdv}{g - \mu v^n}.$$

Then the total distance described (up and down) is

$$h + h_1 = \int_0^V \left(\frac{1}{g - \mu v^n} + \frac{1}{g + \mu v^n}\right) vdv = \int_0^V \frac{2gvdv}{g^2 - \mu^2 v^{2n}},$$

or

$$h + h_1 = \frac{g}{\mu^2}\int_0^V \frac{2vdv}{L^{2n} - V^{2n}} = \frac{g}{\mu^2 L^n}\int_0^V \frac{2vdv}{L^n - (V^2/L)^n}.$$

In this integral, if we let $z = v^2/L$, we shall have

$$h + h_1 = \frac{L^{n+1}}{g}\int_0^{V^2/L} \frac{dz}{L^n - z^n},$$

but this is identical with the integral of (3). Hence $h + h_1 = LT$.

Note.—The incorrectness of the statement of this problem was also pointed out by Professor H. S. Uhler for the special case $n = 2$. However, if in his solution we take the value he finds for h, the height to which the particle rises and the distance z' through which it falls from rest, we have

$$h + z' = \frac{1}{2k}\log\left(1 + \frac{k}{g}V^2\right) + \frac{1}{2k}\log\left(\frac{L^2}{L^2 - V^2}\right) = \frac{1}{2k}\log\left(\frac{L^2 + V^2}{L^2 - V^2}\right),$$

where $\sqrt{g}/\sqrt{k} = V$, the limiting velocity. But he showed that

$$LT = \frac{1}{2k}\log\left(\frac{L^2 + V^2}{L^2 - V^2}\right).$$

Hence, $h + z' = LT$, a result agreeing with that of Professor Reynolds. EDITORS.

NUMBER THEORY.

250. Proposed by JOSEPH E. ROWE, State College, Pa.

Show by comparatively elementary means that the equation $x^{2n} + y^{2n} = z^{2n}$ is impossible of solution in positive integers x, y, z and n, unless at least one of the integers $x, y, z \equiv 0 \pmod 3$. In particular, consider the case $n = 1$.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Since any number must be of one of the forms $3k$, $3k + 1$, $3k - 1$, its square must be of the form $3l$ or $3l + 1$. Consequently, any perfect square, $x_1^{2n} \equiv 0$ or $1 \pmod 3$. It is clear that the three quantities in the given equation can not all be congruent to $1 \pmod 3$; in fact it is evident that either x^{2n} or y^{2n} must be congruent to $0 \pmod 3$, as otherwise the left-hand member of the equation would be divisible by 3 with remainder 2, and the right-hand member by 3 with remainder 1.

The case $n = 1$ may be handled directly by means of the known solution of the equation $x^2 + y^2 = z^2$, namely, $x = 2mn$, $y = m^2 - n^2$, $z = m^2 + n^2$ (we are supposing that x, y and x are prime to each other) where one of the two quantities n, m is even and the other odd. If either m or n is divisible by 3, x is also; if neither, then as we saw above $m^2 \equiv 1 \pmod 3$, $n^2 \equiv 1 \pmod 3$, and, consequently, $y = m^2 - n^2 \equiv 0 \pmod 3$.

Also solved by C. C. YEN, H. C. FEEMSTER, H. N. CARLETON, and J. W. CLAWSON.

251. Proposed by HERMAN ROLAND KATNICK, Chicago, Ill.

Determine the character of the positive integer n so that the Diophantine system

$$z + n = x^2, \qquad z - n = y^2$$

shall have an integral solution; and exhibit a method for finding all the values of x, y, z for a given n of such character.

SOLUTION BY THE PROPOSER.

[Mr. Katnick died suddenly shortly after this problem was offered for publication. The solution given below is a modified and abridged form of one offered by the proposer at the time the problem was submitted. EDITORS.]

From the given equations we have $x^2 - y^2 = 2n$ or $(x + y)(x - y) = 2n$. If we denote $x + y$ by a and $x - y$ by b we have

(1) $$x + y = a, \qquad x - y = b, \qquad 2n = ab.$$

Hence it is necessary that $2x = a + b$, $2y = a - b$, while at least one of the numbers a, b is even (since $ab = 2n$). Hence, both a and b are even since x and y are integers. Then we may put